083/2023

Maximum : 100 marks

Time : 1 hour and 30 minutes

1. Let $V = \mathbb{R}^3$ be a vector space over $\mathbb{R}$ with usual addition and scalar multiplication. Then which of the following is not a subspace of V?

   (A) $W_1 = \{(x_1, x_2, x_3) : x_1 + x_2 = 0, x_1 + 2x_2 - 3x_3 = 0\}$

   (B) $W_2 = \{(x_1, x_2, x_3) : x_1 = 1, x_2 = 1\}$

   (C) $W_3 = \{(x_1, x_2, x_3) : x_1 + x_2 + x_3 = 0\}$

   (D) $W_4 = \{(x_1, x_2, x_3) : x_2 = 3x_1, x_3 = 5x_1\}$

2. Let $(e_1, e_2, ..., e_n\}$ is a standard basis of $\mathbb{R}^n$. Then $\{f_1, f_2, ..., f_n\}$ where $f_1 = a_{11} e_1, f_2 = a_{12} e_2 + a_{22} e_2, .., f_n = a_{1n} e_1 + a_{2n} e_2 + ... + a_{nn} e_n$ is a basis of $\mathbb{R}^n$ if and only if :

   (A) $a_{11} \cdot a_{22} ... a_{nn} = 0$ (B) $a_{11} \cdot a_{22} ... a_{nn} \neq 0$

   (C) $a_{11} + a_{22} + ... + a_{nn} = 0$ (D) $a_{11} + a_{22} + ... + a_{nn} \neq 0$

3. The linear transformation $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^4$ defined by $T(1, 0, 0) = (1, 2, 0, 4)$, $T(0, 1, 0) = (2, 0, -1, -3)$, $T(0, 0, 1) = (0, 0, 0, 0)$. Then which of the following is true?

   (A) $T(y_1, y_2, y_3) = (y_1 + 2y_2, -y_1, y_2, 4y_1 + 3y_2)$

   (B) $T(y_1, y_2, y_3) = (y_1 + 2y_2, 2y_1, -y_2, 4y_1 - 3y_2)$

   (C) $T(y_1, y_2, y_3) = (y_1 - 2y_2, -y_1, y_2, 4y_1 + 3y_2)$

   (D) $T(y_1, y_2, y_3) = (y_1 - 2y_2, y_1, -y_2, -4y_1 - 3y_2)$

4. Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be a linear transformation defined by $T(x_1, x_2, x_3) = (x_1, x_2, 0)$. If r = dimension of range space of T and n = dimension of null space of T, then the value of $2^r + 3^n$ is :

   (A) 5 (B) 7

   (C) 11 (D) 13

5. Let λ be an eigen value of a non-singular matrix A. Then eigen value of adjoint of A, adj A is :

   (A) $|A|$ (B) $\frac{1}{\lambda}$

   (C) 0 (D) $\frac{|A|}{\lambda}$

6. Minimal polynomial of a matrix $M = \begin{bmatrix} 1 & 3 & -2 \\ 0 & 4 & -2 \\ 0 & 3 & -1 \end{bmatrix}$ is :

(A) $(\lambda-1)(\lambda-2)^2$
(B) $(\lambda-1)^2(\lambda-2)$
(C) $(\lambda-1)(\lambda-2)$
(D) $(\lambda-1)^2$

7. Let $A = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a matrix of orthogonal transformation, then its inverse is given by :

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$
(B) $\begin{bmatrix} 0 & 0 & 1 \\ \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \end{bmatrix}$
(C) $\begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$
(D) $\begin{bmatrix} \cos\phi & 0 & \sin\phi \\ -\sin\phi & 0 & \cos\phi \\ 0 & 1 & 0 \end{bmatrix}$

8. Which of the following statement is not true?

(A) Limit of the sequence $\left(1+\frac{1}{n^2}\right)^{n^2}$ is e
(B) Limit of the sequence $\left(1+\frac{1}{n^2}\right)^{2n^2}$ is e^2
(C) Limit of the sequence $\left(1+\frac{1}{2n}\right)^{n}$ is $\sqrt{e}$
(D) Limit of the sequence $\left(1+\frac{2}{n}\right)^{n}$ is e

9. Which of the following function is not uniformly continuous on $(0, \pi)$?

(A) $x \cdot \cos\frac{1}{x}$
(B) e^x
(C) $\frac{e^x}{x}$
(D) $\frac{1}{e^x}$

10. Let $f(x) = |x-1| + |x+1|$ for all $x \in \mathbb{R}$. Then which of the following is true?

(A) f is differentiable at $x = -1$ and $x = 1$
(B) f is differentiable at $x = -1$
(C) f is differentiable at $x = 1$
(D) f is not differentiable at $x = -1$ and $x = 1$

11. By Lagranges Mean value theorem the value of $\sqrt[3]{65}$ lies in the interval :

(A) $\left(4, 4\frac{1}{36}\right)$ (B) $\left(4, 4\frac{1}{24}\right)$

(C) $\left(4, 4\frac{1}{48}\right)$ (D) $\left(4, 4\frac{1}{12}\right)$

12. Let $\{f_n\}$ be a sequence of functions defined by :

(i) $f_n(x) = n\,x\,e^{-nx^2}$, $x \geq 0$ is not uniformly convergent on [0, 1]

(ii) $f_n(x) = \dfrac{x}{1+n\,x^2}$, $x \in \mathbb{R}$ is uniformly convergent on [0, 1]

(A) both (i) and (ii) are true (B) (i) is false and (ii) are true

(C) (i) is true and (ii) is false (D) both (i) and (ii) are false

13. Let g be a bounded function on $[a, b]$. Then which of the following is correct?

(A) If g is a constant function on $[a, b]$, then it is Riemann Integrable on $[a, b]$

(B) If g is continuous on $[a, b]$, then it is Riemann Integrable on $[a, b]$

(C) If g is monotonic on $[a, b]$, then g is Riemann Integrable on $[a, b]$

(D) all of the above

14. Which of the following improper integrals are convergent?

(A) $\displaystyle\int_1^\infty \frac{dx}{\sqrt{x^3+1}}$ (B) $\displaystyle\int_0^\infty \frac{\cos x}{1+x^2}\,dx$

(C) $\displaystyle\int_0^\infty \frac{\cos^2 x}{x^2}\,dx$ (D) all of these

15. Let K be a positive integer and let $J_K = \{x \in [0, 1]:$ decimal expansion of x contains a prime digit at K^{th} place$\}$. Then the Lebesgue measure of the set J_K is :

(A) 1 (B) $\frac{4}{10}$

(C) $\left(\frac{4}{10}\right)^K$ (D) 0

16. Consider $f(x, y) = \begin{cases} \dfrac{x^2 - y^2}{x^2 + y^2}; & if\ (x, y) \neq (0, 0) \\ 0; & if\ (x, y) = (0, 0) \end{cases}$. Then which of the following is true?

(A) f is continuous at (0, 0)

(B) f is bounded in a neighbourhood of (0, 0)

(C) f is not bounded in a neighbourhood of (0, 0)

(D) f has all directional derivatives at (0, 0)

17. Let $f, g : [0, 1] \to \mathbb{R}$ defined by $f(x) = \begin{cases} x, & if\ x = \frac{1}{n}, n \in \mathbb{N} \\ 0, & otherwise \end{cases}$ $g(x) = \begin{cases} x, & if\ x \in Q \cap [0, 1] \\ 0, & otherwise \end{cases}$ then :

(A) both f and g are Riemann Integrable
(B) f is Riemann Integrable, g is Lebesgue Integrable
(C) g is Riemann Integrable, f is Lebesgue Integrable
(D) Neither f nor g is Riemann Integrable

18. Let $f : \mathbb{R}^3 \to \mathbb{R}^3$ is given by $f(y_1, y_2, y_3) = (e^{y_2} \cos y_1, e^{y_2} \sin y_1, 2y_1 - \cos y_3)$ and let $E = \{(y_1, y_2, y_3) \in \mathbb{R}^3 :$ there exists an open set V around (y_1, y_2, y_3) such that the restriction f/v is open map$\}$. Then which of the following is true?

(A) $E = \mathbb{R}^3$
(B) E is countable
(C) $\left\{y_1, y_2, \frac{\pi}{2}\right\} \in \mathbb{R}^3 : y_1, y_2 \in \mathbb{R}\}$ is a proper subset of E
(D) All of the above are true

19. The value of $\int_0^\infty e^{-x^4} dx$ is :

(A) $\frac{1}{2}\Gamma\frac{1}{2}$
(B) $\frac{1}{3}\Gamma\frac{1}{3}$
(C) $\frac{1}{4}\Gamma\frac{1}{4}$
(D) $\frac{1}{5}\Gamma\frac{1}{5}$

20. The value of $\int_0^1 x^3(1-x^2)^4 dx$ is :

(A) $\frac{1}{2}\beta(2, 5)$
(B) $\frac{1}{3}\beta(3, 4)$
(C) $\frac{1}{4}\beta(2, 3)$
(D) $\frac{1}{2}\beta(2, 3)$

21. The Fourier series corresponding to the function $f(x) = \begin{cases} -3, & -5 < x < 0 \\ 3, & 0 < x < 5 \end{cases}$ contains :

(A) cosine terms only
(B) sine terms only
(C) both sine and cosine terms
(D) sine terms and non-zero constant terms

22. Let G be a group with binary operation $*$ defined by $a * b = a + 2b - 3$ for all $a, b \in G$. Then the inverse of the element $c \in G$ is :

(A) $\frac{2c+9}{4}$
(B) $\frac{9-2c}{4}$
(C) $\frac{c-3}{4}$
(D) $\frac{c+2}{4}$

23. In the finite group $G = \{2, 4, 6, 8\}$ with respect to multiplication modulo 10, the identity element is :

(A) 2
(B) 4
(C) 6
(D) 8

24. Which of the following is true?

(i) Every group of order 42 has a normal subgroup of order 7
(ii) Every group of order 42 has a normal subgroup of order 8

(A) both (i) and (ii) are true
(B) neither (i) nor (ii) are true
(C) only (i) is true
(D) only (ii) is true

25. Every group of even permutations in S_n can be written as a product of ———— number of transpositions:

(A) $2n+1$
(B) $2n+3$
(C) $2n+4$
(D) $2n+8$

26. Consider the statements :

(i) Number of homomorphisms from $\mathbb{Z}_{10}$ to $\mathbb{Z}_{25}$ is 5
(ii) Number of homomorphisms from $\mathbb{Z}_{100}$ to $\mathbb{Z}_{92}$ is 4
(iii) Number of homomorphisms from $\mathbb{Z}_{16}$ to $\mathbb{Z}_{19}$ is 2
(iv) Number of non-trivial homomorphisms from $\mathbb{Z}_{13}$ to $\mathbb{Z}_{19}$ is 1

(A) all the statements are true
(B) only (i) and (ii) are true
(C) only (i), (ii) and (iii) are true
(D) only (i), (ii) and (iv) are true

27. The largest possible order of any element in Aut ($\mathbb{Z}_{200}$) is :

(A) 200
(B) 100
(C) 20
(D) 1

28. Consider the statements :

(i) No group of order 96 is simple
(ii) No group of order 160 is simple

Then

(A) Neither (i) nor (ii) is true
(B) Both (i) and (ii) are true
(C) (i) is true and (ii) is false
(D) (i) is false and (ii) is true

29. Which of the following is true?

(i) The set $S = \left\{ \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} : a, b \in \mathbb{Z} \right\}$ is a subring of the ring of M_2 of 2 × 2 matrices over integers
(ii) An element $x \in R$ is called idempotent if $x^2 = x$
(iii) Let R be a ring such that $x^3 = x$ for all $x \in R$, then R is commutative
(iv) Let R be a ring of 2×2 matrices over reals, then $S = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \text{ is a real number} \right\}$ is not a subring of R

(A) All the statements are true
(B) Only (i), (ii) and (iii) are true
(C) Only (i) and (ii) are true
(D) Only (ii) and (iii) are true

30. Consider the statements :

(i) The set $R = \{a + b\sqrt{3} : a, b \in \mathbb{Q}\}$ is a field under usual addition and multiplication

(ii) The set $S = \left\{ \begin{pmatrix} x & y \\ -\bar{x} & \bar{y} \end{pmatrix} : x, y \in \mathbb{C} \right\}$ is a division ring

(iii) The set $\mathbb{C} = \{a + ib : a, b \in \mathbb{R}\}$ of complex numbers is a field under usual addition and multiplication of complex numbers.

(iv) The set $\mathbb{Z}_4$ is an integral domain

Then

(A) all statements are true

(B) only (i), (ii) and (iii) are true

(C) only (i) and (iii) are true

(D) only (i) is true

31. Let R be the ring of all real valued continuous functions of the closed unit interval. Then which of the following are maximal ideals of R :

(i) $M_1 = \left\{ f \in R : f\left(\frac{1}{5}\right) = 0 \right\}$

(ii) $M_2 = \left\{ f \in R : f\left(\frac{2}{3}\right) = 0 \right\}$

(iii) $M_3 = \left\{ f \in R : f\left(\frac{1}{3}\right) = 0 \right\}$

Then

(A) M_1, M_2 and M_3 are maximal ideals

(B) M_1 and M_3 are maximal ideals

(C) Only M_2 is maximal ideal

(D) M_1, M_2 and M_3 are not maximal ideals

32. Let PID, ED, UFD denotes the set of all principal ideal domain, euclidean domain, unique factorization domain respectively. Then :

(A) UFD $\subset$ ED $\subset$ PID

(B) PID $\subset$ ED $\subset$ UFD

(C) ED $\subset$ PID $\subset$ UFD

(D) PID $\subset$ UFD $\subset$ ED

33. Which of the following is not true?

(A) $\sqrt{-3}$ is a prime element of $\mathbb{Z}[\sqrt{-3}]$

(B) $2 + \sqrt{-5}$ is irreducible but not prime in $\mathbb{Z}[\sqrt{-5}]$

(C) $(1 - i)$ is an irreducible element of $\mathbb{Z}[i]$

(D) For any prime p, the ring $\mathbb{Z}_p$ of integers modulo p is a PID

34. The degree of the splitting field $x^5 - 3x^3 + x^2 - 3$ over $\mathbb{Q}$ is :

(A) 2
(B) 3
(C) 4
(D) 8

35. Let G be the Galois group of the splitting field of $x^5 - 2$ over $\mathbb{Q}$. Then :

(i) G is cyclic,
(ii) G is non-abelian
(iii) G has order 20
(iv) G has an element of order 4

(A) All (i), (ii), (iii) and (iv) are true
(B) (i), (ii) and (iii) are true
(C) (ii), (iii) and (iv) are true
(D) (i), (iii) and (iv) are true

36. Let τ be the topology on $\mathbb{N}$ (natural numbers) consists of φ and all subsets of $\mathbb{N}$ of the form $A_n = \{n, n+1, n+2, ...\}$, $n \in \mathbb{N}$. The set of all accumulation points of the set $A = \{8, 21, 33, 55\}$ is :

(A) {1, 2, 3, ...7}
(B) {8, 9, ..., 55}
(C) {1, 2, 3, ...54}
(D) {56, 57, ...}

37. Let X be a non-empty set, under which of the following topologies τ_1 and τ_2 on X the identity function, $I = (X, \tau_1) \to (X, \tau_2)$ is continuous?

(i) τ_1 and τ_2 are only two topologies on X
(ii) τ_1 - indiscrete topology and τ_2 - discrete topology
(iii) τ_1 - any topology, τ_2 - indiscrete topology
(iv) τ_1 - discrete topology, τ_2 - any topology

(A) all of the above (i), (ii), (iii) and (iv)
(B) only (iv)
(C) only (ii) and (iii)
(D) only (iii) and (iv)

38. In the set $\mathbb{R}$ with infinite topology, the sequence $\left(\frac{1}{n}\right)$, $n \in \mathbb{N}$ converges to :

(A) 0 only
(B) all real numbers
(C) the numbers $1, \frac{1}{2}, \frac{1}{3}, ...$ only
(D) 0 and 1 only

39. Which of the following set is dense in $\mathbb{R}$ under the topology on $\mathbb{R}$ consisting of $\mathbb{R}$, φ and all open intervals of the form (x, ∞), $x \in \mathbb{R}$?

(A) {–10, –5, 0, 5, 10,...}
(B) {–10, –9, –8,...,9, 10}
(C) {...–6, –3, 0, 3}
(D) {10, 12, 20}

40. Which of the following is not metrizable?

(i) $\mathbb{R}$ with cofinite topology

(ii) $\mathbb{R}$ with topology consists of $\mathbb{R}$, φ and all open intervals of the form (x, ∞), $x \in \mathbb{R}$

(iii) $\mathbb{R}$ with indiscrete topology

(A) (i) only
(B) (ii) only
(C) (i) and (ii) only
(D) all of the above (i), (ii) and (iii)

41. Let X be a countable set in $\mathbb{R}$. The subspace topology on X induced by the usual topology in $\mathbb{R}$ is:

(A) connected
(B) compact
(C) not second countable
(D) normal

42. Consider the metrics on $\mathbb{R}^2$ given by $d_1(x, y) = \begin{cases} 1 & if \quad x \neq y \\ 0 & if \quad x = y \end{cases}$

$d_2(x, y) = |x_1 - y_1| + |x_1 - y_2|$ and

$d_3(x, y) = \max\{|x_1 - y_1|, |x_2 - y_2|\}$ for $x = (x_1, x_2)$ and $y = (y_1, y_2)$ in $\mathbb{R}^2$.

Then :

(A) only d_1 and d_2 are equivalent
(B) only d_1 and d_3 are equivalent
(C) only d_2 and d_3 are equivalent
(D) d_1, d_2 and d_3 are equivalent

43. The positive integer n for which $(1+i)^n = 512$ is :

(A) 12
(B) 8
(C) 18
(D) 16

44. The value of the complex number i^{-2i} is :

(A) $e^{(4n+1)\pi}$ for $n \in \mathbb{Z}$
(B) $e^{-(4n+1)\pi}$ for $n \in \mathbb{Z}$
(C) $\frac{i\pi}{2}$
(D) $\frac{-i\pi}{2}$

45. The function $f(z) = |z|^2$ for $z \in \mathbb{C}$ is:

(A) analytic at 0
(B) differentiable at 0
(C) differentiable at all non-zero complex numbers
(D) analytic at all non-zero complex numbers

46. If f and $\bar{f}$ are both analytic on a region D in the complex plane, then :

(A) $f(z) = z$ for all $z \in D$
(B) $f(z) = c$ for all $z \in D$ and some constant c
(C) $f(z) = \bar{z}$ for all $z \in D$
(D) $f(z) = |z|$ for all $z \in D$

47. If c is the circle $|z|=1$, then $\oint_C \frac{(z+1)}{z^4+2iz^3}dz$ is :

(A) $\frac{-\pi}{4}+i\frac{\pi}{2}$
(B) $3\pi+6\pi i$
(C) $\frac{\pi}{4}-i\frac{\pi}{2}$
(D) $\frac{\pi}{2}-i\frac{\pi}{4}$

48. If c is the circle $|z|=1$, then $\oint_C e^{\frac{3}{z}}dz$ is :

(A) $2\pi i$
(B) $3\pi i$
(C) 6
(D) $6\pi i$

49. Let G be an open connected set in $\mathbb{C}$ such that G contains an interval X of the real line and let $f, g : G \to \mathbb{C}$ be analytic. If $f(z)=g(z)$ for all $z \in X$, then for $z \in G$, $g(z)$ is :

(A) $g(z)$ is real valued
(B) $g(z)$ is constant
(C) $g(z) = z$
(D) $g(z) = f(z)$

50. $F : \mathbb{R}^5 \to \mathbb{R}$ is the bounded linear map $F(x_1, x_2, \ldots x_5) = x_1 + 2x_2 + 7x_3 + 3x_4 + x_5$. If $\mathbb{R}^5$ is bestowed with the 2-norm $\| \; \|_2$, then $\|F\|$ is :

(A) 8
(B) 14
(C) 42
(D) 64

51. Which of the following is a Banach space?

(i) c_{oo} as a subspace of l^∞
(ii) $c[a, b]$ as a subspace of $L^p[a, b]$, $1 \le p < \infty$
(iii) $c[a, b]$ with supremum norm
(iv) $c'[a, b]$ with supremum norm

(A) (i) and (iii)
(B) (iii) only
(C) (iii) and (iv)
(D) (ii) only

52. The adjoint of the bounded linear map $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (2x - y, 3x - y)$ is :

(A) $T^*(x, y) = (2x+3y, -x-y)$
(B) $T^*(x, y) = (5x-2y, x-y)$
(C) $T^*(x, y) = (-x+2y, -x+3y)$
(D) $T^*(x, y) = (x-2y, 2x-y)$

53. Which of the following is a continuous linear functional on an inner product space X over $\mathbb{C}$. For $x \in X$, :

(i) $f(x) = \|x\|$
(ii) $f(x) = \langle x, y \rangle$ for some $y \in x$
(iii) $f(x) = \langle y, x \rangle$ for some $y \in x$

(A) (i)
(B) (ii)
(C) (ii) and (iii)
(D) (i) and (ii)

54. Which of the following metrics in $\mathbb{R}^2$ is not induced by a norm in $\mathbb{R}^2$. For $x=(x_1,x_2)$ and $y=(y_1, y_2)$ in $\mathbb{R}^2$?

(A) $d_1(x, y)=\begin{cases}1 & if \quad x\neq y\\ 0 & if \quad x=y\end{cases}$

(B) $d_2(x, y)=|x_1-y_1|+|x_2-y_2|$

(C) $d_3(x, y)=\max\{|x_1-y_1|, |x_2-y_2|\}$

(D) $d_4(x, y)=\sqrt{(x_1-y_1)^2+(x_2-y_2)^2}$

55. Let M be an orthonormal set in an inner product space X. If d is the metric induced by the norm, then for distinct points $x, y\in M, d(x, y)$ is :

(A) $d(x, y)=0$

(B) $d(x, y)=1$

(C) $d(x, y)=2$

(D) $d(x, y)=\sqrt{2}$

56. The Hahn-Banach extension of $h(x, y)=2x$ on $Y=\{(x, y\in\mathbb{R}^2/x=y\}$ to $\mathbb{R}^2$ with 2-norm is :

(A) $f(x, y)=3x-y$

(B) $f(x, y)=x+y$

(C) $f(x, y)=4x-2y$

(D) $f(x, y)=-x+3y$

57. Solution of $(\tan^{-1}y-x)\,dy=(1+y^2)\,dx$ is :

(A) $x=\tan^{-1}y-1+c\ e^{\tan^{-1}y}$

(B) $y\ \tan^{-1}y=c\ e^{\tan^{-1}x}$

(C) $y\ \sec x=x\tan^{-1}y+cx^2$

(D) None of the above

58. Solution of $(1+x)^2\dfrac{d^2y}{dx^2}+(1+x)\dfrac{dy}{dx}+y=2\sin[\log(1+x)]$ is :

(A) $y=c_1\cos x+c_2\sin x+\log(1+x)\cos x$

(B) $y=c_1\cos[\log(1+x)]+c_2\sin[\log(1+x)]-\log(1+x)\cos[\log(1+x)]$

(C) $y=c_1e^{2x}+c_2e^{3x}\log(1+x)\sin(1+x)$

(D) $y=c_1\cos[\log(1+x)]+c_2\sin[\log(1+x)]+\cos[\log(1+x)]$

59. If $p(x)$ is a continuous function, the general solution of :

$y''-p(x)\,y'+[p(x)-1]\,y=0$ is :

(A) $y=c_1e^x+c_2e^xe^{\int p(x)\,dx}$

(B) $y=c_1e^x+c_2e^x\int p(x)\,dx$

(C) $y=c_1e^x+c_2\int e^{\int p(x)\,dx}dx$

(D) $y=c_1e^x+c_2e^x\int e^{[-2x+\int p(x)\,dx]}dx$

60. Consider the equation $y'=y^{\frac{1}{3}}, y(0)=0$ on $X=\{(x, y\}\in\mathbb{R}^2/|x|\leq 1, |y|\leq 2\}$, Which of the following is true?

(A) differential equation has unique solution

(B) $f(x, y)=y^{\frac{1}{3}}$ satisfies Lipschitz condition with respect to y in X

(C) differential equation has no solutions

(D) none of the above

61. The equation $\dfrac{dy}{dx}=2y^{\frac{1}{2}}$, $y(0)=0$ has :

(A) Unique solution

(B) Infinite number of solutions

(C) No solution

(D) Not unique but finite number of solutions

62. Solutions of the partial differential equation $2xz - px^2 - 2qxy + pq = 0$ (where $p = \frac{\delta z}{\delta x}, q = \frac{\delta z}{\delta y}$) is :

(A) $z = c_1 x^2 + c_2 y(x - c_1)$
(B) $z = c_1 x + c_2 x^2 y$
(C) $z = c_1 y + c_2 (x^2 - c_1)$
(D) $z = c_1 xy + c_2 y^2$

63. Solution of $\frac{\delta^2 z}{\delta x^2} + 4\frac{\delta^2 z}{\delta x \, \delta y} - 5\frac{\delta^2 z}{\delta y^2} = \sin(2x + 3y)$ is :

(A) $z = \varphi_1(y + x) + \varphi_2(y - 5x) + \frac{1}{17}\sin(2x + 3y)$

(B) $z = \varphi_1(y - x) + \varphi_2(y + 5x) + \frac{1}{17}\cos(2x + 3y)$

(C) $z = \varphi_1(y + 2x) + \varphi_2(y - 3x) + \frac{1}{17}\sin(2x - 3y)$

(D) $z = \varphi_1(y + x) + \varphi_2(y - 5x) - \frac{1}{4}\sin(x - 5x)$

64. The remainder when 3^{181} is divided by 17 is :
(A) 1
(B) 7
(C) 11
(D) 5

65. The last two digits of 3^{1000} is :
(A) 11
(B) 01
(C) 36
(D) 29

66. If $1 + 7 + 7^2 + \ldots + 7^{17} \equiv a(\text{mod}\,19)$, then a is :
(A) 7
(B) 11
(C) 0
(D) 1

67. The number of positive integers less than or equal to 2076 and divisible by neither 4 nor 5 is :
(A) 1156
(B) 1245
(C) 982
(D) 886

68. The ones digit in the decimal value of 17^{6666} is :
(A) 7
(B) 5
(C) 9
(D) 4

69. Which of the following number is divisible by 7?
(A) 526262625
(B) 29295286126
(C) 125689721
(D) 291889767890

70. The number of incongruent solutions of $155x \equiv 75(\text{mod}\,65)$ is :
(A) 5
(B) 13
(C) 8
(D) 15

71. The main objective/s of learning is/are :

(i) To gather information about student weaknesses.

(ii) Modification of behaviour.

(iii) To adopt innovative methods of teaching.

(iv) To identify the areas of further improvement in teaching learning process.

(A) (i) and (ii) only
(B) (ii), (iii) and (iv) only
(C) (i), (ii) and (iii) only
(D) (ii) only

72. The evaluation to assess the overall effectiveness of a program and grade the pupil is :

(A) Diagnostic evaluation
(B) Formative evaluation
(C) Summative evaluation
(D) Continuous evaluation

73. Backward exploration is a shortcut that makes the process of problem solving quick and efficient. What does backward exploration mean?

(A) It is a process in which we analyse the previous solutions of a problem, apply them to the problem and check which is relevant one.

(B) It refers to envisioning the end or ultimate goal to determine the best strategy to achieve a goal by solving a problem.

(C) A set of rules to be followed to perform a specific task.

(D) A cognitive bias that limits the use of an object in a particular way.

74. What is the sequence of steps of lesson planning?

(i) Followup activity

(ii) Developmental activity

(iii) Setting of specific objectives

(iv) Introductory activity

(v) Recapitulation

(A) (i), (ii), (iii), (iv), (v)
(B) (ii), (iii), (iv), (v), (i)
(C) (iii), (iv), (ii), (v), (i)
(D) (iv), (ii), (i), (iii), (v)

75. At which of the following stages of Piaget's theory of cognitive development is a child confronted with confusing ideas?

(A) Concrete operational stage
(B) Sensorimotor stage
(C) Formal operational stage
(D) Preoperational stage

76. Match the statement given in the List A with the List B :

	List A		List B
(i)	Research by the user for the user	(1)	Experimental research
(ii)	Research to establish the existence of cause-and-effect relationship between two variables	(2)	Fundamental research
(iii)	Research for adding new knowledge to the existing repository of knowledge	(3)	Action research
		(4)	Applied research

	(i)	(ii)	(iii)
(A)	(3)	(2)	(1)
(B)	(4)	(3)	(1)
(C)	(3)	(1)	(2)
(D)	(4)	(1)	(2)

77. Which one of the following refers to the authenticity or genuineness of the document?

(A) External Criticism
(B) External Validity
(C) Internal Validity
(D) Internal Criticism

78. Which of the following is the correct sequence of steps in historical research?

(i) Identify primary and secondary data sources.

(ii) Analyze the data and develop a narrative exposition of the findings.

(iii) Evaluate the authenticity and accuracy of source materials.

(iv) Collection of data

(A) (i), (iv), (iii), (ii)
(B) (ii), (iii), (iv), (i)
(C) (ii), (iv), (i), (iii)
(D) (iii), (i), (iv), (ii)

79. Which of the following options most appropriately explains 'Research Ethics'?

(A) It states how to write a research report with stylistic language.

(B) To avoid presenting other's work as own

(C) Description of methodology

(D) It provides a common set of do's and don'ts of conducting ethical research

80. An academic lecture or presentation to an audience on certain topics of educational nature, it is called :

(A) Conference (B) Workshop

(C) Seminar (D) Symposium

81. The Constitution of India exhibits federal characteristics, eventhough it does not explicitly define India as a federal state. Which of the following statements are TRUE about the federal characteristics of the Indian Constitution?

(i) The Constitution provides for an independent and impartial judiciary.

(ii) The Constitution establishes institutional checks and balances to limit the power of both states and the centre.

(iii) The central government possesses more powers than state governments, making the constitution more unitary in nature.

(A) Only (i) and (ii) (B) Only (ii) and (iii)

(C) Only (i) and (iii) (D) All the above (i), (ii) and (iii)

82. The Indian Constitution includes a comprehensive chapter on fundamental rights. Which of the following statements regarding fundamental rights in the Indian Constitution is/are TRUE?

(i) The right to life and liberty prohibits discrimination based on religion, race, caste, sex, or birthplace.

(ii) Individuals can approach the judiciary in India to enforce their fundamental rights under the right to constitutional remedy.

(iii) Cultural and educational rights protect the interests of minorities by preserving their language, script and culture.

(A) Only (i) and (ii) (B) Only (ii) and (iii)

(C) Only (i) and (iii) (D) All the above (i), (ii) and (iii)

83. Which of the following is NOT true about the Indian judiciary?

(A) It is responsible for interpreting the Constitution

(B) It consists of the Supreme Court and several High Courts in different states

(C) It serves as a check on the powers of the executive and legislative branches of the government

(D) The doctrine of "due process of law" gives the judiciary more power than the legislature.

84. Find the appropriate match between the colunms of constitutional Amendments in India and the Goals of Amendment in the following table. Then, select the correct answer group :

	Constitutional Amendments		Goals of Amendment
(i)	102nd Amendment	(1)	The Bill aims to give State governments the power to identify socially and educationally backward OBCs
(ii)	103rd Amendment	(2)	To extend the reservation of seats for Scheduled Castes and Scheduled Tribes in the Lok Sabha and State Assemblies until 2030
(iii)	104th Amendment	(3)	To reserve seats for economically weaker sections of society in government educational institutions and public employment.
(iv)	105th Amendment	(4)	Create a National Commission for Backward Classes and grant the President the authority to identify socially and educationally disadvantaged groups.

	(i)	(ii)	(iii)	(iv)
(A)	(1)	(2)	(3)	(4)
(B)	(2)	(1)	(4)	(3)
(C)	(3)	(2)	(1)	(4)
(D)	(4)	(3)	(2)	(1)

85. Though instituted much later, the fundamental duties are an inevitable part of the Indian Constitution. Which of the following statement is/are TRUE about the fundamental duties enshrined in the Constitution?

(i) They are a set of moral and ethical obligations that every citizen of India is expected to fulfil.

(ii) These are not legally enforceable but promote responsibility and patriotism among citizens.

(iii) They cover a specific range of areas and are limited to duties related to the Constitution, the national flag and anthem.

(iv) They are complementary to the fundamental rights enshrined in the Indian Constitution.

(A) Only (i), (ii) and (iii)

(B) Only (ii), (iii) and (iv)

(C) Only (i), (ii) and (iv)

(D) All the above (i), (ii), (iii) and (iv)

86. Which of the following is NOT one of the effects of the emergency provisions mentioned in the Indian Constitution?

(A) The President can issue ordinances that have the same force as Acts of Parliament.

(B) The life of the Rajya Sabha can be extended by one year at a time.

(C) The Centre can give directions to the state governments on any matter.

(D) The fundamental rights of citizens can be suspended during a national emergency.

87. The Constitution of India contains an exclusive provision for amendment. Which of the following statements is TRUE about the amendment procedure in the Constitution of India?

(i) Amendments related to the admission or establishment of new states can be made by a simple majority of the two houses of Parliament.

(ii) The amendment procedure balances the need for flexibility with the need for stability and continuity.

(iii) The parliament has initiated the "basic structure" doctrine of the Constitution, which cannot be amended.

(A) Only (i) and (ii)
(B) Only (ii) and (iii)
(C) Only (i) and (iii)
(D) All the above (i), (ii) and (iii)

88. Which of the following statements about the Directive Principles of State Policy (DPSP) in the Indian Constitution is NOT true?

(A) The DPSPs are meant to guide the government in its policy-making and decision-making.

(B) The Directive Principles aim to improve people's welfare by incorporating socio-economic and political conditions into all aspects of life.

(C) The DPSPs are enforceable by any court in India.

(D) The DPSP seeks to establish economic and social democracy in the country.

89. Which among the following are the functions of the NHRC?

(i) To enquire complaints of violation of human rights or negligence in the prevention of violation by a public servant

(ii) To study human rights treaties and international instruments, and recommend their effective implementation to the government.

(iii) To spread human rights awareness and encourages efforts in human rights literacy at national and international levels.

(iv) To hear complaints related to service matters and matters that are sub-judice.

(A) Only (i), (ii) and (iii)
(B) Only (ii), (iii) and (iv)
(C) Only (i), (iii) and (iv)
(D) All the above (i), (ii), (iii) and (iv)

90. Which among the following are the functions of the Union Public Service Commission?

(i) Conducting interviews and assessments for various military positions

(ii) Conducting competitive examinations for various civil services positions.

(iii) Selecting candidates for various civil services positions based on merit and performance

(iv) Advising the government on matters related to personnel management and recruitment

(A) Only (i), (ii) and (iii)

(B) Only (ii), (iii) and (iv)

(C) Only (i), (iii) and (iv)

(D) All the above (i), (ii) (iii) and (iv)

91. Which statement is/are incorrect regarding the Ramon MagSaySay Award?

(i) It is known as the "Nobel Prize of Asia".

(ii) Dalailama is the first recipient of this Award as spiritual leader.

(iii) Blessed Mother Teresa is a recipient of this Award in 1986.

(iv) Dr. M.S. Swaminathan is not a recipient of this Award.

(A) (i) and (ii) only

(B) (i) and (iii) only

(C) (ii) and (iii) only

(D) (iii) and (iv) only

92. ICC Men's T20 world cup held in November 2022. Which statement is/are correct regarding the final match?

(i) It was played at Melbourne Cricket Ground on 13th November 2022.

(ii) England won the match by 5 wickets.

(iii) Kumar Dharmasena was an on-field umpire in the match.

(iv) Joe Root of England was the 'Player of the Match'.

(A) (i) and (ii) only

(B) (i), (ii) and (iii) only

(C) (i), (ii) and (iv) only

(D) (ii), (iii) and (iv) only

93. Write the chronological order of the formation of the following socio-religious reform movements of Kerala :

(i) Islam Dharma Paripalana Sangham.

(ii) Vaala Samudaya Parishkarani Sabha.

(iii) Samathwa Samajam

(iv) Sadhu Jana Paripalana Sangham.

(A) (i), (ii), (iii), (iv)

(B) (ii), (iii), (iv), (i)

(C) (iii), (i), (iv), (ii)

(D) (iv), (iii), (i), (ii)

94. Which statement is/are incorrect regarding the Paliyam Sathyagraha?

(i) It was officially inaugurated by Panampilly Govinda Menon.

(ii) It was the first post independent Satyagraha organized in Kerala.

(iii) It was supported by women from all parts of Kerala.

(iv) A.G. Velayudham was killed in police lathi charge.

(A) (i) only

(B) (ii) only

(C) (iii) only

(D) (iv) only

95. The Abstention Movement was a historic event in Travancore history. From the following statements, find out the wrong statement/statements :

(i) It was a joint venture of the Ezhava, Muslim and Christian Communities.

(ii) C. Kesavan, delivered the famous 'Kozhencherry Speech' on May 11, 1934.

(iii) Franchise was widened by reducing the property qualification.

(iv) It brought the issue of responsible government in Travancore.

(A) (i) only

(B) (ii) only

(C) (i) and (ii) only

(D) (iii) and (iv) only

96. Match the following :

(i)	Rajyasamacharam	(1)	Moorkoth Kumaran
(ii)	Vivekodayam	(2)	Swami Guruprasad
(iii)	Gajakesari	(3)	Basel Mission Society
(iv)	Mithavadi	(4)	Ezhava Gazette

	(i)	(ii)	(iii)	(iv)
(A)	(2)	(1)	(4)	(3)
(B)	(3)	(4)	(1)	(2)
(C)	(4)	(1)	(2)	(3)
(D)	(3)	(4)	(2)	(1)

97. Find out the incorrect statement/statements from the following :

(i) Bhaskara Menon is regarded as the first detective novel in Malayalam.

(ii) Kerala Sangeetha Nataka Academy was inaugurated by Pandit Jawaharlal Nehru, the then Prime Minister of India, on 26 April 1958.

(iii) M.K. Sanu was the President of Purogamana Kala Sahithya Sangham during 1988-90.

(iv) Lilathilakam is a 16^{th} Century treatise on Malayalam grammar and poetics.

(A) (i) only

(B) (ii) and (iv) only

(C) (i) and (iii) only

(D) (iv) only

98. Match the following :

(i)	Parvathy Nenmenimangalam	(1)	Agnipushpangal
(ii)	Lalithambika Antharjanam	(2)	KPCC President in 1944
(iii)	A.V. Kuttimalu Amma	(3)	Founder and editor of the journal "Shreemati"
(iv)	Justice Anna Chandy	(4)	Antharjana Samajam

	(i)	(ii)	(iii)	(iv)
(A)	(4)	(1)	(2)	(3)
(B)	(3)	(4)	(1)	(2)
(C)	(2)	(4)	(3)	(1)
(D)	(1)	(3)	(4)	(2)

99. Find out the wrong pair from the following table :

(i)	Dr. Ayyathan Gopalan	–	Rao Sahib
(ii)	K.P. Vallon	–	'Adhakrithan' magazine
(iii)	P.K. Chathan Master	–	Member of Legislative Assembly from Chelakkara
(iv)	T.K. Krishnaswami Iyer	–	Called 'Untouchable Brahmin'

(A) (i) only
(B) (ii) only
(C) (iii) only
(D) (iv) only

100. Match the following :

(i)	P.Kesavadev	(1)	Ente Vazhiyambalangal
(ii)	Ponkunnam Varkey	(2)	Ethirppu
(iii)	S.K. Pottekkatt	(3)	Kazhinjakalam
(iv)	K.P. Kesava Menon	(4)	Ente Vazhithirivu

	(i)	(ii)	(iii)	(iv)
(A)	(4)	(1)	(2)	(3)
(B)	(2)	(4)	(1)	(3)
(C)	(3)	(2)	(4)	(1)
(D)	(2)	(1)	(4)	(3)

SPACE FOR ROUGH WORK

SPACE FOR ROUGH WORK