Syllabus

Assistant Professor in Statistics- Collegiate Education

I. Probability Theory

Concepts of probability, independence of events, addition theorem, multiplication theorem, conditional probability and Bayes theorem. Discrete and continuous random variables, distribution function, density function, probability generating function, moment generating function, characteristic function, inversion formula and uniqueness theorem, expectation of random variables and its properties, moments, cumulants.

Convergence: Modes of convergence, Convergence in probability, in distribution, in rth mean, almost sure convergence and their inter-relationships.

Laws of Large Numbers: Kolmogorov's inequality, Weak law of large numbers, (Khinchine's and Kolmogorov's), Kolmogorov's strong law of large numbers, Central limit theorems(CLT) for i.i.d random variables, Lindberg-Levy CLT, Liaponov's CLT, Lindberg-Feller CLT.

II. Statistical Estimation Theory

Point estimation: Sufficiency and minimal sufficiency, Exponential family of distributions, Factorization criterion. Unbiased estimation, Completeness, Ancillary statistics and Basu's theorem, UMVUE estimators and their characterizations, Rao-Blackwell Theorem, Lehmann Scheffe Theorem, Fisher information measure and its properties, Cramer-Rao inequality. Consistent estimators, mean square error, CAN estimator.

Methods of estimation: Methods of moments, Maximum likelihood estimation, Least square estimation and their properties.
Bayesian Estimation: Loss function, prior distribution, Bayes theorem, posterior distributions, Bayes risk, Bayes principle, Bayes estimators.

Interval Estimation: Large sample confidence intervals, UMA and UMAU confidence intervals, Shortest expected length confidence intervals, construction of confidence intervals using pivots.

III. Testing of hypothesis

Fundamental concepts of testing of hypothesis, Neyman-Pearson lemma, UMP tests, unbiased tests and UMPU tests, unbiased critical regions and similar regions, likelihood ratio tests, construction of sequential probability ratio tests, Wald's identity, OC and ASN functions, Properties of SPRT.

Non-parametric inference: Goodness of fit tests- Chi square test and Kolmogorov Smirnov test, Sign test, Signed rank test, Run test, Median test, Mann-Whitney U-test, Wilcoxan test, Kruskall Wallis test

IV. Design of Experiments

Basic principles of design of experiments, Gauss Markoff Theorem, CRD, RBD and LSD, Balanced Incomplete Block designs, Partially balanced incomplete block designs, Lattice designs, Analysis of co-variance for completely randomized and randomized block designs. Analysis of experiments with missing observations.

Factorial Designs: Principles, factorial Designs (2 factor, General factor, 2^k factorial), Confounding techniques- Partial and Total, fractional factorial design.

V. Sampling Theory

Population, sample, sampling design, simple random sampling (SRS with replacement and without replacement, stratified random sampling, systematic sampling, cluster sampling, ratio estimators, regression estimators, probability proportional to size with replacement sampling, varying probability without replacement (Midzuno-Sen-Lahiri sampling strategy, Desraj, Horvic's Thompson, Murthy's), multiphase and mutistage sampling.

VI. Distribution Theory

Univariate random variables, distribution functions and their properties, bivariate distributions- joint, marginal and conditional distribution, independence of random variables.

Discrete distributions- binomial, Poisson, negative binomial, geometric, uniform, hyper geometric, power series and multinomial distributions.

Continuous distributions- uniform, normal, exponential, double exponential, beta, gamma, Cauchy, Weibull, Pareto, log-normal and logistic distributions, Pearson system of distributions.

Order Statistics- Joint, marginal and conditional distributions of order statistics of a random sample arising from continuous distributions, distribution of sample median and range

Sampling distributions- distribution of the mean and variance of a random sample from normal population, Chi-square, t, and F distributions.

VII. Multivariate Analysis

Concepts of multivariate distributions, multivariate normal distribution(MND), marginal and conditional distributions, characteristic function of MND, estimation of mean vector and covariance matrix, Wishart distribution and its properties, distribution of simple, partial and multiple correlations based on samples from normal population, Hotelling's T^2 and Mahalanobis D^2 statistics and their properties, multivariate Fisher Behren's problem, testing independence of sets of variates, testing equality of covariance matrices and means, sphericity tests, testing the hypothesis that a covariance matrix equal to given matrix.

Classification problem, procedures of classification into one of two populations with known probability distributions, classification into one of two known multivariate normal populations, classification into one of several populations, principal component analysis and its properties, canonical variables and canonical correlations, factor analysis.

VIII. Stochastic Process

Concept of Stochastic processes, examples, specifications, Markov chains, transition probability matrix, Chapman Kolmogorov equations, recurrence, transience and periodicity of states, classification of states, limiting probabilities, stationary distribution, ergodic theorem.

Gamblers ruin problem and random Walk, branching processes (discrete time), probability of ultimate extinction.

Continuous time Markov Chains: counting process – inter arrival time and waiting time distributions. Poisson processes and properties, conditional distribution of arrival times, Generalization of Poisson processes (non-homogenous Poisson process, compound Poisson process, conditional mixed Poisson process), birth and death processes and applications to queueing models, Markovian queues, M/G/1, G/M/1 queues, multi server queues.

Renewal processes, renewal theorems and their applications, renewal reward process, regenerative processes, Brownian motion processes.

IX. Regression Analysis

Linear Regression Model, least squares estimation, Gauss Markov setup, properties of the estimates, maximum likelihood estimation, generalised least squares, Hypothesis testing - likelihood ratio test, F-test. Confidence intervals.

Residual analysis, collinearity, non-constant variance and serial correlation, departures from normality, diagnostics and remedies

Polynomial regression in one and several variables, orthogonal polynomials, indicator variables, subset selection of explanatory variables, stepwise regression.
Nonlinear regression, estimation and diagnosis methods for logistic and Poisson regressions.

X. Mathematical Methods for Statistics

Riemann-Stieltjes Integral- definition, existence and properties, integration by parts, change of variable - Step functions as integrators. Reduction to finite sum, Improper integrals- Gamma and Beta integrals.

Point wise convergence and uniform convergence of sequences and series of functions.

Multivariable functions, limit and continuity of multivariable functions, derivatives, directional derivatives and continuity, total derivative in terms of partial derivatives, optima of multivariable functions.

Elementary matrices, determinants, rank of a matrix, inverse, diagonal reduction, transformations, idempotent matrices, solution of linear equations, characteristic roots and vectors, Cayley- Hamilton theorem, algebraic and geometric multiplicity of characteristic roots, spectral decomposition, quadratic forms, classification and reduction of quadratic forms, generalized inverse, Moore-Penrose inverse- definition and existence, properties of generalized inverse.
