

66/2015

(Pages : 4)

Maximum : 200 marks

Time : 1½ hours

**PART I**

Answer ALL questions. Each question carries 4 marks.

1. If  $P(A) = 0.2$ ,  $P(B) = 0.3$  and  $P(A \cap B) = 0.1$ , find  $P(A/B)$  and  $P(B/A)$ .
2. What do mean by Cramer - Rao Lower bound?
3. Let  $X$  be a random variable with distribution function :

$$F(x) = 0, x \leq 0 \\ = 1 - e^{-\lambda x}, x > 0$$

Obtain the moment generating function.

4. In a business venture a man can make profit of ₹ 2,000/- with a probability of 0.4 and a loss of ₹ 1,000/- with a probability of 0.6. What is his expected profit?
5. A continuous random variable  $X$  has a probability density function  $f(x) = 3x^2$ ,  $0 < x < 1$ , find 'a' such that  $P(X \leq a) = P(X > a)$ .
6. What are the applications of Chi- square distribution?
7. What are the characteristics of a good estimator?
8. State Naymann-Pearson Lemma.
9. State Central limit theorem.
10. Distinguish between parameter and statistic.
11. Define F Statistic and write its p. d. f.

[P.T.O.]

12. A random variable  $X$  has mean 3 and variance 2. Use Tchebychev's inequality to obtain an upper bound for  $P\{|x-3| \geq 2\}$ .
13. If  $x$  is a random variable distributed as  $N(0,1)$ , show that  $Y = X^2$  has gamma distribution.
14. Let  $X$  has the density  $f(x) = 1, 0 < x < 1$ , find the distribution of  $-2 \log x$ .
15. In the analysis of data of randomized block design with 5 blocks and 4 treatments, what is the degrees of freedom for the error variations?
16. What are the basic assumptions of ANOVA?
17. The two regression lines are  $X + 2Y = 5$  and  $2X + 3Y = 8$ . Variance of  $Y = 4$ . What is the value of variance of  $X$ ?
18. Differentiate between estimator and estimate.
19. What are the properties of distribution function of a random variable?
20. Write down the characteristic function of a normal distribution.
21. If the first four raw moments of a random variable  $X$  are 1,2,3 and 4 respectively, find the first four central moments.
22. What is the relationship between  $t$ -distribution and  $F$ -distribution?
23. For the sequence of random variables  $\{X_n\}$  with  $P\{X_k = k^k\} = P\{X_k = -k^k\} = \frac{1}{2}$ . Test whether weak law of large numbers holds for  $\{X_n\}$ .
24. How will you test the significance of correlation coefficient  $r$ ?
25. Find the coefficient of skewness if the difference between third and first quartiles is 8, sum of these quartiles is 22 and the median is 10.5.
26. Variance  $(x) = 25$  and mean  $(x) = 308$ , find the coefficient of variation.

27. Define level of significance and power of test.
28. Write down the test statistic for testing equality of two distributions using Kolmogorov Smirnov Test.
29. Show that the series  $1 + \frac{1}{1!} + \frac{1}{2!} + \dots$  is convergent.
30. What is the probability that out of 10 deliveries in a hospital on a day 5 results in male births?

(30 × 4 = 120)

## PART II

Answer ALL questions. Each question carries 10 marks.

31. Find the characteristic function of Laplace distribution with pdf  $f(x) = \frac{1}{2}e^{-|x|}$ .
32. In a sample of 1000 people in Maharashtra 540 are rice eaters and the rest are wheat eaters. Can we assume that rice eaters and wheat eaters are equally popular in this state at 1% level of significance?
33. X and Y are two random variables having the joint density function,  $f(x, y) = \frac{x+2y}{27}$ , where x and y assumes values 0, 1 and 2. Find the conditional distribution of Y given  $X = x$ .
34. Find the maximum likelihood estimate for the parameter  $\lambda$  of a Poisson distribution on the basis of sample of size n. Also find its variance.
35. If  $X \geq 1$  is the critical region for testing  $\lambda = 2$  against the alternative  $\lambda = 1$ , on the basis of a single observation from a population,  $f(x, \lambda) = \lambda e^{-\lambda}$ ,  $0 < x < \infty$ . Obtain the value of Type I Error and Type II error.
36. Explain the three basic principles of experimental design.

37. Let  $X$  be non-negative continuous random variable with distribution function  $F(x)$ , show

$$\text{that } E(X) = \int_0^{\infty} [1 - F(x)] dx.$$

38. Let  $f$  be a function on  $\mathbb{R}$  defined by

$$f(x) = 1, \text{ when } x \text{ is rational}$$

$$= -1, \text{ when } x \text{ is irrational}$$

Show that  $f$  is discontinuous at every point of  $\mathbb{R} \int_0^{\infty} [1 - F(x)] dx.$

(8 × 10 = 80)